11th Serbia Conference on Spectral Line Shapes in Astropysics
(Sabac, Serbia, August 21-25, 2017)

M.T. Meftah, K. Arif, K. Chenini, K. Touati, and S. Douis

Laboratoire de Recherche sur la Physique des Plamas et des Surfaces (LRPPS) et Département de Physique, Faculté de Mathématiques et Sciences de la Matière, Université Kasdi-Merbah de Ouargla 30000, Algérie

*Contribution Of Lienard-Wiechert Potential To The Electron Broadening Of Spectral Line Shapes in Plasmas**

- * *Contribution Of Lienard-Wiechert Potential In The Electron Broadening*

Plan of the representation

I- Introduction

II- Theoretical Basic of the Electron Broadening

III- Discussion and Results

IV- Conclusion

I- Introduction

- Line profiles and shifts are used to determine plasma parameters, especially in astrophysics where alternative methods (such as interferometry or Thomson scattering) are not possible.
- In a number of hot astrophysical plasmas, electrons may be energetic enough that their thermal energy KBT can be comparable to the rest mass.
- For the extreme densities encountered in some astrophysical objects, pressure broadening could dominate; however for such objects the electrons may become relativistic due to the extreme temperatures and hence it makes sense to check the modifications to the pressure broadening by relativistic effects.
- In the present work we investigate the region corresponding to the particular conditions of plasma: high density and high temperature. Under these conditions, (electron-ion) collisions will be, throughout this work, assumed binary and the dynamics of the electrons will be treated relativistically.
- we focus on electron broadening in the impact approximation. We thus revisit the standard semiclassical collision operator and take into account relativistic effects with respect to the trajectory.
- In many cases in line broadening, fast particles(typically electrons) are described by a collisional approach, while particles whose field variation on the inverse half width half maximum (HWHM) time scale are considered static and treated via a quasistatic microfield. For many applications, isolated lines are of great importance.
- Calculations of the broadening of such a line in a plasma are normally made using the impact approximation for electrons in the semi-classical version, as the ionic contribution is typically negligible.
- We only consider isolated lines, since within the impact approximation the collision operator for more complex cases is basically expressible in terms of this case.
- This work is organized in four sections. The second section derives the collision operator along the lines of the standard nonrelativistic framework, but with account for relativistic effects. The third section compares the relativistic and nonrelativistic operators and discusses the obtained results, while the last section coclude the results.

II- THEORETICAL BASIC OF THE ELECTRON BROADENING

•In high ionization plasmas the Stark effects is dominant. The broadening of this effects tends to be important only for lines from the electric dipole transitions allowed.

•The two dominant approximations in the computations of the electronic collision operator Φ are the dipole approximation and the approximation of the classical path which considers the perturbed electrons I impact approximation.

•Our calculation is based on the same approximations, or we will compute the expression of the electronic collision operator in the relativistic case Φ *.

•The generalized formula of the profile of the line according to the relativistic contribution is defined by:

$$I(\omega) = -\frac{1}{\pi} Re\left\{\vec{d}_{\alpha\beta} \langle\langle \alpha\beta \left| \left(i\omega - \frac{i(H_g - H_e)}{\hbar} + \Phi^*_{eg} \right)^{-1} \right| \alpha'\beta' \rangle\rangle \vec{d}^*_{\alpha'\beta'} \right\} \quad (1)$$

• Φ^*_{eg} is the operator of relativistic electronic collisions, independent of time and ionic microfield.

- To calculate this operator we begin with the general expression:

$$\langle\langle\alpha\beta\left|\left(\Phi_{eg}^{*}\right)^{-1}\right|\alpha'\beta'\rangle\rangle = \sum_{\alpha''}\vec{r}_{\alpha\alpha''}\,\vec{r}_{\alpha''\alpha'}\Phi_{d}^{*}(\omega_{\alpha\alpha''},\omega_{\alpha''\alpha'}) + \sum_{\beta''}\vec{r}_{\beta\beta''}\,\vec{r}_{\beta''\beta'}\Phi_{d}^{*}(\omega_{\beta\beta''},\omega_{\beta''\beta'}) - \vec{r}_{\alpha\alpha'}\,\vec{r}_{\beta'\beta}\Phi_{int}^{*}(\omega_{\alpha\alpha'},\omega_{\beta'\beta}) \qquad (2)$$

- Where $\vec{r}_{ab}$ is the matrix element of the position operator of the electron.
- We calculate the direct relativistic term Φ_{d}^{*} and relativistic term of interferene Φ_{int}^{*}
- The study of Φ_{d}^{*} is carried out in the case of the isolated lines for an ion transmitter and hyperbolic paths for the free electrons.
- The formula of the relativistic collision operator is :

$$\Phi_{d}^{*}(\omega_1,\omega_2) = -\frac{2\pi N_e e^2}{3\hbar^2}\int_0^c v f(\beta)\,d\beta\int_{\rho_{min}}^{\rho_{max}}\rho_0\,d\rho_0\int_{-\infty}^{+\infty}dt_1\int_{-\infty}^{t_1}dt_2\,e^{(i\omega_1 t_1, i\omega_2 t_2)}\left[\vec{E}_{LW}(t_1).\vec{E}_{LW}(t_2)\right] \qquad (3)$$

- Where $f(\beta)\,d\beta$ is the Juttner- Maxwell distribution function $f(\beta)\,d\beta = \frac{\gamma^5\beta^2 d\beta}{\theta K_2(1/\theta)}exp(-\gamma/\theta)$
- $\theta = \frac{K_B T}{m_e c^2}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $K_2(1/\theta)$ is a Bessel function, ρ_0 is the non-relativistic impact

parameter, where $\vec{E}_{LW}(\vec{R},t)$ is the electric field of Lienard-Wiechert given by:

$$\vec{E}_{LW}(\vec{R},\vec{\beta},t) = -e\frac{(\vec{n}-\vec{\beta})(1-\beta^2)}{k^3R^2(t')} - \frac{e}{c^2}\frac{\vec{n}}{k^3R(t')}\times\left\{(\vec{n}-\vec{\beta})\times\frac{d\vec{v}(t')}{dt'}\right\} \qquad (4)$$

$$\vec{\beta} = \frac{\vec{v}(t')}{c}, \qquad \vec{\eta} = \frac{\vec{R}(t')}{R(t')}, \; t' = t - R(t')/c$$

were v is the initial velocity of perturbed electron and c is the speed of light in the vacuum.

Where the retarded time is given by: $t' = t - R(t')/c$

e is the charge of the electron, and $R(t')$ is the electron position vector, and $\vec{\eta} = \frac{\vec{R}((t')}{R(t')}$ is a unit vector directed from the position of the charge to observation point, and k is given by:

$$k = \frac{dt}{dt'} = \frac{1}{c}\frac{dR(t')}{dt'} = 1 + \vec{\eta}.\vec{\beta}$$

the first part of the field, the velocity field (4), goes to the ordinary Coulomb field when $v \rightarrow 0$
And the second part of field, the acceleration field is called the radiation field.
We can neglect the second term of E_{LW} and use only the first part of the field in the subsequent development

$$\vec{E}_{LW}(\vec{R},t) \simeq e\left[\frac{(\vec{\eta} - \vec{\beta})(1-\beta^2)}{k^3 R^2}\right] \qquad (5)$$

We use the approximation $1 - \beta^2 \simeq 1$ therefore the elctric field becomes

$$\vec{E}_{LW}(\vec{R},t) \simeq e\left[\frac{(\vec{\eta} - \vec{\beta})}{k^3 R^2}\right] \qquad (6)$$

We neglect the fine structure, we can write the collision operator amplitude as :

$$\Phi_d^*(0,0) = -\frac{\pi N_e e^2}{3\hbar^2}\int_0^c v f(\beta)\, d\beta \int_{\rho_{min}}^{\rho_{max}} \rho_0 d\,\rho_0 \int_{-\infty}^{+\infty} \overrightarrow{E_{LW}}(t_1)\,dt_1 \int_{-\infty}^{+\infty} \overrightarrow{E_{LW}}(t_2)\,dt_2 \qquad (7)$$

After more calculation and simplifications, we can obtain the final formula of the amplitude of the collision operator as :

$$\Phi_d^*(0,0)$$

$$= \left(-\frac{4N_e e^4}{3\pi\hbar^2}\frac{1}{\theta c K_2(1/\theta)}\right)\int_0^1 \frac{exp\left(\frac{-1}{\theta\sqrt{1-\beta^2}}\right)}{(1-\beta^2)^{\frac{5}{2}}}\beta d\beta \int_{\epsilon_{min}}^{\epsilon_{max}} \frac{[(\epsilon^2-1)-\beta^2\epsilon^2\alpha]^2}{\epsilon(\epsilon^2-1)\alpha^6} d\epsilon \qquad (8)$$

We transform now the integral over β to over γ we find

$$\Phi_d^*(0,0) \simeq \left(-\frac{4N_e e^4}{3\pi\hbar^2}\frac{1}{\theta c K_2(1/\theta)}\right)\int_0^1 \frac{exp\left(\frac{-1}{\theta\sqrt{1-\beta^2}}\right)}{(1-\beta^2)^{\frac{5}{2}}}\beta d\beta \int_{\epsilon_{min}}^{\epsilon_{max}} \frac{\epsilon^2-1}{\epsilon\alpha^4} d\epsilon \qquad (9)$$

$$= \left(-\frac{4N_e e^4}{3\pi\hbar^2}\frac{1}{\theta c K_2(1/\theta)}\right)\int_0^1 \frac{exp\left(\frac{-1}{\theta\sqrt{1-\beta^2}}\right)}{(1-\beta^2)^{\frac{5}{2}}}\beta d\beta \int_{\epsilon_{min}}^{\epsilon_{max}} \frac{\epsilon^2-1}{\epsilon(1-\epsilon^2+\beta^2\epsilon^2)} d\epsilon \qquad (10)$$

$$\Phi_d^*(0,0) = \left(-\frac{4N_e e^4}{3\pi\hbar^2}\frac{1}{\theta c K_2(1/\theta)}\right)\int_1^\infty \gamma^6 exp\left(\frac{-\gamma}{\theta}\right) d\gamma \int_{\epsilon_{min}}^{\epsilon_{max}} \frac{\epsilon^2-1}{\epsilon(\gamma^2-\epsilon^2)^2} d\epsilon \qquad (11)$$

Now we must to express ϵ_{min} and ϵ_{max} as function of γ

$$\epsilon_{max}^2 = 1 + q^2 \lambda_D^2 \left(\frac{\gamma^2 - 1}{\gamma^2} \right)^2 \qquad (12)$$

$$\epsilon_{min}^2 = 1 + q^2 a_0^2 \left(\frac{\gamma^2 - 1}{\gamma^2} \right)^2 \qquad (13)$$

We obtain finally

$$\Phi_d^{*}(0,0)\left({}^{Hz}/_{cm^2} \right) = \left(-\frac{4N_e e^4}{3\pi \hbar^2} \frac{1}{\theta c K_2\left({}^{1}/_{\theta} \right)} \right) \int_1^{\infty} \gamma^2 exp\left(\frac{-\gamma}{\theta} \right) d\gamma \left[ln\left(\frac{\epsilon_{min}^2 - \gamma^2}{\epsilon_{max}^2 - \gamma^2} \right) - 2ln \frac{\epsilon_{min}}{\epsilon_{max}} + \frac{1 - \gamma^2}{\gamma^{-2}\, \epsilon_{min}^2 - 1} - \frac{1 - \gamma^2}{\gamma^{-2}\, \epsilon_{max}^2 - 1} \right] \qquad (14)$$

It remains the integral over γ, that we have calculate numerically the amplitude of the collision operator.

Diapositive 8

F8 Il faut une diapositive sur la théorie, avant de prsenter les résultats et les discussions.
Fethi; 20/05/2011

III- Discussion and Results

In this work, we have investigated Lienard-Wiechert or retarded electric elds produced by moving electric charges with respect to a rest frame. Specially, we have look its contribution to the broadening of the spectral line shape in hot and dense plasmas.

At T=3,2.10^9K we observe that the classical and the relativistic operator collision are in te same order, for the densitie of 10^{18} cm^{-3} the difference between the classical and relativistic is in 0.05eV (28.26%).
A difference of 0.01eV (7.14%) between the classical and relativisti operator for the densitie of 10^{22} cm^{-3}
The difference decrease with the increasing of densities (table1).

$N_e(cm^{-3})$	Φ_C(eV)	Φ^*_R(eV)	Percent(%)
10^{16}	0.24367e-6	0.15e-6	38.16
10^{18}	0.2091e-4	0.15e-4	28.26
10^{20}	0.17453e-2	0.16e-2	8.3
10^{22}	0.13995	0.15	7.14

Table 1: Comparaison between the calssical and relativistic operator collision for Ag^{+46} T=3,2.10^9K

$$\text{percent} = \frac{\Phi_C - \Phi^*_R}{\Phi_C} \times 100$$

IV- Conclusion

In this work, we have investigated Lienard-Wiechert or retarded electric fields produced by moving electric charges with respect to a rest frame. Specially, we have look its contribution to the broadening of the spectral line shape in hot and dense plasmas.

THANKS FOR YOUR ATTENTION